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Transient Thermal Response of a Homogeneous Composite Thin Layer Exposed to a Fluctuating Heating Source under the Effect of the Dual-Phase-Lag Heat-Conduction Model

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The transient thermal behavior of a homogeneous composite domain described by three macroscopic heat-conduction models, under the effect of a fluctuating heating source, was investigated analytically. The composite domain consists of a matrix (domain 1) and inserts (domain 2), each made of different material. The matrix has a high concentration or volume fraction (>0.5) while the insert has a low concentration or volume fraction (<0.5). The range of parameters within which the use of the hyperbolic or the dual-phase-lag heat-conduction models is a necessity was traced. The role that the frequency and amplitude of the fluctuating thermal disturbance plays in using the appropriate macroscopic heat-conduction model was studied.

KEY WORDS: composite; dual-phase-lag model; heat conduction; fluctuating heat source.

1. INTRODUCTION

The thermal behavior of a composite thin layer exposed to a fluctuating heating source is an important problem in heat transfer. It is used to simulate a wide range of applications such as laser synthesis and processing of thin-film deposition [1]. Such applications involve a heat source such as a laser and/or microwave of extremely short duration or very high frequency, very high temperature gradients, and extremely short times; and the heat

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is found to propagate at a finite speed. Since the classical Fourier heat flux model is based on an infinite speed of propagation (heat flux and temperature gradients occur at the same time), it should be modified to account for the finite speed of propagation.

Cattaneo [2] and Vernotte [3] suggested independently a modified heat flux model. The constitutive law assumes that the heat flux vector (the effect) and the temperature gradient (the cause) across a material volume occur at different instants of time and the time delay between the heat flux and the temperature gradient is the relaxation time $\overline{\tau}_q$. This leads to the classical hyperbolic heat-conduction equation (HHCE) [4]:

$$\frac{1}{\alpha}\frac{\partial T}{\partial t}+\frac{\overline{\tau}_q}{\alpha}\frac{\partial^2 T}{\partial t^2}=\nabla^2 T+\frac{g}{k}+\frac{\overline{\tau}_q}{k}\frac{\partial g}{\partial t} \tag{1}$$

To remove the precedence assumption made in the thermal wave model, as proposed in the HHCE, the dual-phase-lag (DPL) model is proposed [5–7]. The dual-phase-lag model allows either the temperature gradient (cause) to precede the heat flux vector (effect) or the heat flux vector (cause) to precede the temperature gradient (effect) in the transient process. Mathematically, the heat-conduction equation under the dual-phase-lag effect [5–7] is expressed as

$$\begin{aligned}\frac{1}{\alpha}\frac{\partial T}{\partial t}(t,\vec{r})+\frac{\overline{\tau}_q}{\alpha}\frac{\partial^2 T}{\partial t^2}(t,\vec{r}) &= \nabla^2 T(t,\vec{r})+\overline{\tau}_T\frac{\partial}{\partial t}\left[\nabla^2 T(t,\vec{r})\right]\\ &\quad+\frac{1}{k}\left[g+\overline{\tau}_q\frac{\partial g}{\partial t}(t,\vec{r})\right]\end{aligned} \tag{2}$$

where $\overline{\tau}_T$ is the phase lag in the temperature gradient vector and $\overline{\tau}_q$ is the phase lag in the heat flux vector. For the case of $\overline{\tau}_T > \overline{\tau}_q$, the temperature gradient established across a material volume is a result of the heat flow, implying that the heat flux vector is the cause and the temperature gradient is the effect. For $\overline{\tau}_T < \overline{\tau}_q$, on the other hand, heat flow is induced by the temperature gradient established at an earlier time, implying that the temperature gradient is the cause, while the heat flux vector is the effect.

In the absence of the temperature gradient phase lag ($\overline{\tau}_T = 0$), Eq. (2) reduces to the classical hyperbolic heat-conduction equation as described by Eq. (1). Also, in the absence of the two phase lags ($\overline{\tau}_T = \overline{\tau}_q = 0$), Eq. (2) reduces to the classical diffusion equation employing Fourier's law.

In the literature, numerous studies have been conducted to investigate the thermal behavior of slabs subject to non-fluctuating heating sources under the effect of the hyperbolic and dual-phase-lag heat-conduction models. A comprehensive survey of work in this area can be found in Ref. 8. However, all of the previous investigations deal with a homogeneous domain consisting of a single material.

The thermal behavior of a multi-layered thin slab under the effect of the dual-phase-lag heat-conduction model has been investigated [9, 10]. But in many applications the structure of the film consists of a one-layer composite materiel [11]. Only a few studies [12–14] have been conducted to investigate the dynamic thermo-elastic behavior of a composite slab using the hyperbolic and dual-phase-lag heat-conduction models, but in these studies the heating source considered was a unit step function.

The aim of the present work is to investigate the thermal behavior of a homogeneous composite domain heated by a very high frequency fluctuating heating source. The composite domain consists of a well-mixed dominant matrix (domain 1) and inserts (domain 2), each made of different material. The matrix has a high concentration or volume fraction (>0.5) while the insert has a low concentration or volume fraction (<0.5). The thermal behavior of such a composite domain will be investigated as described by the three macroscopic heat-conduction models and under the effect of a fluctuating heating source with very high frequency. The range of parameters within which the use of the hyperbolic or the dual-phase-lag heat-conduction models is a necessity will be traced. The role that the frequency of the fluctuating thermal disturbance plays in using the appropriate macroscopic heat-conduction model will be studied.

2. ANALYSIS

Consider a composite layer of thickness $2L$ for which the boundaries are maintained at a fixed temperature T_{W}. The homogeneous composite layer consists of a well-mixed dominant matrix (domain 1) and inserts (domain 2) as shown in Fig. 1. The inserts may be added to reinforce the slab's structure or to give the slab certain required physical and chemical properties, or may be impurities formed during the production process. A high frequency volumetric heating source generates heat within the matrix domain. The second domain, the inserts, is set to be a stationary solid, without heat generation ($u_2 = 0$).

At this step of the analysis, the following dimensionless parameters are introduced:

$$\theta = \frac{T - T_\infty}{T_\infty}, \quad \eta = \frac{\alpha_1 t}{L^2}, \quad \xi = \frac{x}{L}, \quad \text{and} \quad \tau_j = \frac{\overline{\tau}_j \alpha_1}{L^2},$$

The governing equations describing the slab thermal behavior can be written in the following dimensionless form [14]:

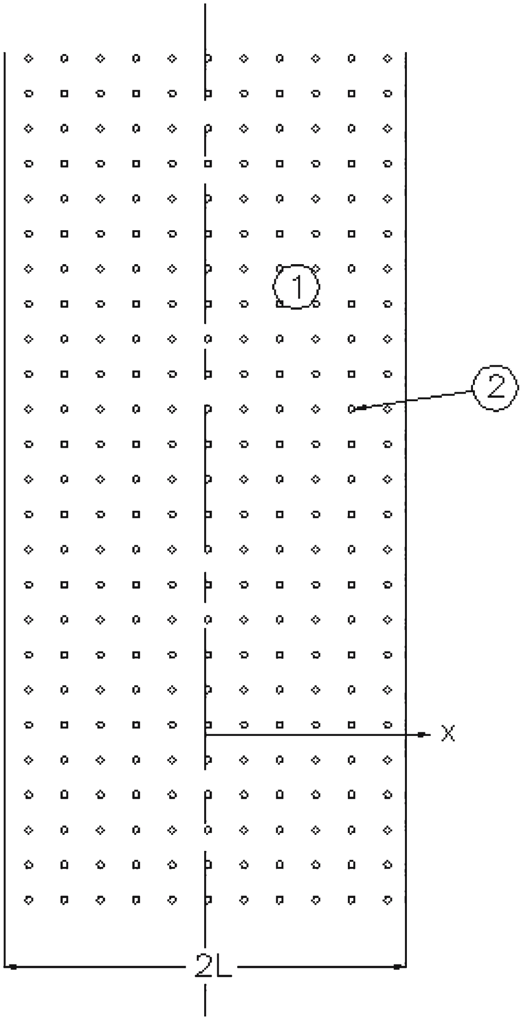

Fig. 1. Schematic representation of the domain under consideration.

$$\tau_{q,1}\frac{\partial^2\theta_1}{\partial\eta^2}+\frac{\partial\theta_1}{\partial\eta}+\tau_{q,1}E\left(\frac{\partial\theta_1}{\partial\eta}-\frac{\partial\theta_2}{\partial\eta}\right)$$
$$=\frac{\partial^2\theta_1}{\partial\xi^2}+\tau_{T,1}\frac{\partial^3\theta_1}{\partial\eta\partial\xi^2}+U_1+E\left(\theta_2-\theta_1\right)+\tau_{q,1}\frac{\partial U_1}{\partial\eta} \tag{3a}$$

$$\tau_{q,2}\frac{\partial^2\theta_2}{\partial\eta^2}+\frac{\partial\theta_2}{\partial\eta}+\tau_{q,2}F\left(\frac{\partial\theta_2}{\partial\eta}-\frac{\partial\theta_1}{\partial\eta}\right)$$
$$=B\frac{\partial^2\theta_2}{\partial\xi^2}+\tau_{T,2}B\frac{\partial^3\theta_2}{\partial\eta\partial\xi^2}+F\left(\theta_1-\theta_2\right) \tag{3b}$$

$$E=\frac{hL^2}{(1-\varepsilon)\,k_1},\quad F=\frac{hL^2}{k_1}\frac{\rho_1c_1}{\varepsilon\rho_2c_2},\quad B=\frac{k_2}{k_1}\frac{\rho_1c_1}{\rho_2c_2}.$$

where ε is the insert volume fraction $\varepsilon=V_1/(V_1+V_2)$, V is the volume, ρ is the mass density, c is the specific heat, h is the volumetric heat transfer coefficient that represents all modes of heat transfer between the matrix and the inserts, and subscripts 1 and 2 refer to domain 1 (matrix) and domain 2 (inserts), respectively.

It will be assumed that thermal diffusion within the inserts (domain 2) has an insignificant effect due to the fact that these inserts are in the form of discrete, infinitesimal regions that are not in perfect thermal contact with each other. In addition, neglecting thermal diffusion within the second domain is justified when this domain consists of low thermal conductivity impurities. As a result of one or both of these two assumptions, the conduction within the inserts is neglected and B is set to zero.

Two special heat-conduction models may be obtained from Eq. (3). These are the hyperbolic (wave) heat-conduction model which is obtained by setting $\tau_{T_1}=\tau_{T_2}=0$, and the parabolic (diffusion) heat-conduction model which is obtained by setting

$$\tau_{q_1}=\tau_{q_2}=\tau_{T_1}=\tau_{T_2}=0.$$

Since the composite domain is homogeneous and well-mixed, it has the following boundary conditions:

$$\begin{aligned}\theta_1(\eta,1)&=\theta_2(\eta,1)=0\\ \frac{\partial\theta_1}{\partial\xi}(\eta,0)&=\frac{\partial\theta_2}{\partial\xi}(\eta,0)=0\end{aligned} \tag{4}$$

3. SOLUTION METHODOLOGY

A fluctuating harmonic heating source is assumed to be generated within the main domain (matrix), and it has the following form:

$$U_1=U_o\mathrm{Im}\left\{e^{i\omega\eta}\right\} \tag{5}$$

where ω is the dimensionless frequency given as $\omega = \frac{\overline{\omega} L^2}{\alpha}$ and $\overline{\omega}$ is the angular velocity of the fluctuating heating source. Although the heating source may fluctuate in another form than sinusoidal, it can be expressed in terms of a summation of trigonometric harmonic functions using a Fourier series expansion. With $B = 0$, Eqs. (3a) and (3b) are rewritten as

$$\tau_{q,1}\frac{\partial^2\theta_1}{\partial\eta^2} + \frac{\partial\theta_1}{\partial\eta} + \tau_{q,1}E\left(\frac{\partial\theta_1}{\partial\eta} - \frac{\partial\theta_2}{\partial\eta}\right) = \frac{\partial^2\theta_1}{\partial\xi^2} + \tau_{T,1}\frac{\partial^3\theta_1}{\partial\eta\partial\xi^2}$$
$$+U_o\mathrm{Im}\left\{e^{i\omega\eta}\right\} + E\left(\theta_2 - \theta_1\right) + \tau_{q,1}U_o i\omega\mathrm{Im}\left\{e^{i\omega\eta}\right\} \tag{6a}$$

$$\tau_{q,2}\frac{\partial^2\theta_2}{\partial\eta^2} + \frac{\partial\theta_2}{\partial\eta} + \tau_{q,2}F\left(\frac{\partial\theta_2}{\partial\eta} - \frac{\partial\theta_1}{\partial\eta}\right) = F\left(\theta_1 - \theta_2\right) \tag{6b}$$

with the following boundary conditions:

$$\theta_1(\eta, 1) = \theta_2(\eta, 1) = 0$$
$$\frac{\partial\theta_1}{\partial\xi}(\eta, 0) = \frac{\partial\theta_2}{\partial\xi}(\eta, 0) = 0 \tag{7}$$

It should be noted here that since the heating source is fluctuating, no initial condition is needed. Since the temperature within the two domains will have to follow the heating source in its general profile, even though not with the same amplitude, θ can be assumed to be in the form,

$$\theta_j = \mathrm{Im}\left\{V_j\left(\xi\right)e^{i\omega\eta}\right\} \tag{8}$$

Substituting this form into Eq. (6) and solving, we obtain the following solution:

$$V_1\left(\xi\right) = \frac{\gamma}{\lambda^2}\left(1 - \frac{\cosh\lambda\xi}{\cosh\lambda}\right) \tag{9}$$

$$V_2 = \beta V_1 \tag{10}$$

where

$$\gamma = U_o\frac{1 + i\omega\tau_{q,1}}{1 + i\omega\tau_{T,1}} \tag{11}$$

$$\lambda^2 = \frac{E - \tau_{q,1}\omega^2 + i\omega + E\tau_{q,1}i\omega - E\beta - E\beta\tau_{q,1}i\omega}{1 + \tau_{T,1}i\omega} \tag{12}$$

$$\text{and } \beta = \frac{F + \tau_{q,2}Fi\omega}{F - \tau_{q,2}\omega^2 + i\omega + \tau_{q,2}Fi\omega} \tag{13}$$

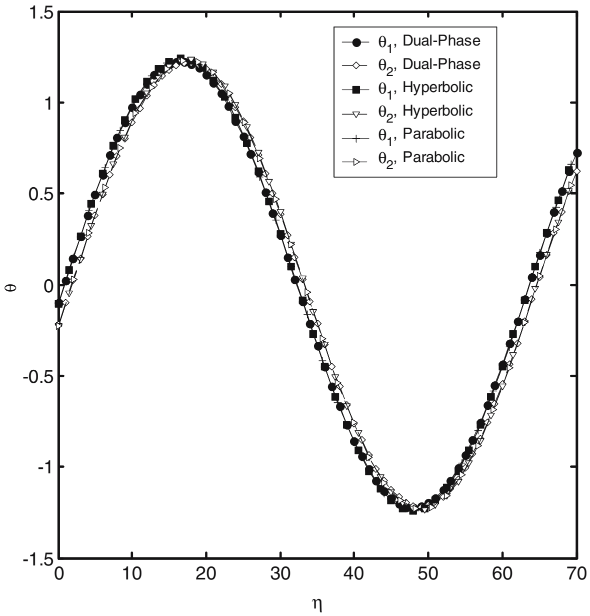

Fig. 2. Harmonic variation in temperature as predicted by the three models. ($\omega = 0.1, \xi = 0.5, \tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}, \tau_{q,1} = 1 \times 10^{-2}, E = F = 1$).

It is clear from Eq. (10) that β represents the deviation in magnitude and the phase shift in the time domain between θ_1 and θ_2. This equation can also be written as

$$V^2 = \sqrt{a^2 + b^2} e^{i\delta} \tag{14}$$

where

$$a = F - \tau_{q,2}\omega^2 + \tau_{q,2}\omega\left(\omega + \tau_{q,2}F\omega\right) \tag{15}$$

$$b = \tau_{q,2}\omega\left(F - \tau_{q,2}\omega^2\right) - \omega - \tau_{q,2}F\omega \tag{16}$$

$$\text{and } \delta = \tan^{-1}\left(\frac{b}{a}\right) \tag{17}$$

where δ represents the phase shift between θ_1 and θ_2, which gives an indication about the time required by θ_2 to follow the changes in θ_1.

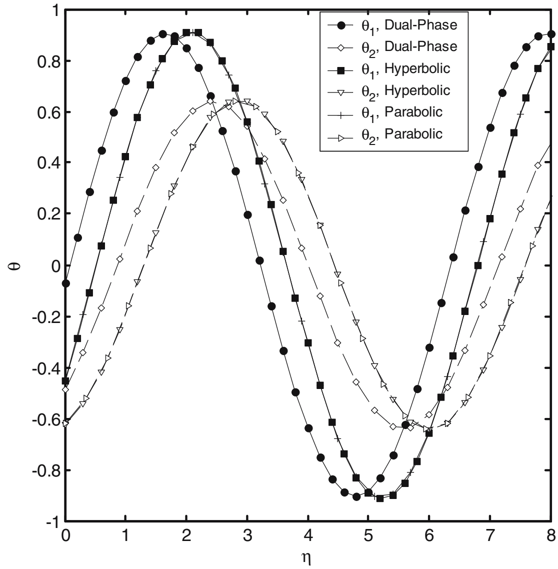

Fig. 3. Harmonic variation in temperature as predicted by the three models. ($\omega = 1.0$, $\xi = 0.5$, $\tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}$, $\tau_{q,1} = 1 \times 10^{-2}$, $E = F = 1$).

4. RESULTS AND DISCUSSION

It is clear from the governing equations that the dimensionless parameters that affect the composite domain thermal behavior are E, F, $\tau_{q,1}$, $\tau_{q,2}$, $\tau_{T,1}$, ω, and U_o. The parameters E and F represent a sort of modified volumetric heat transfer coefficients. These parameters measure the convective heat transfer coefficients between the hot matrix (domain 1) and the inserts (domain 2). The parameters $\tau_{q,1}$ and $\tau_{q,2}$ represent the relaxation times in heat flux in both domains, while the parameter $\tau_{T,1}$ represents the relaxation time in the temperature gradient in domain 1. It is clear that $\tau_{T,2}$ does not appear in the present work because the diffusion heat flux in domain 2 is assumed to be insignificant. This is due to the fact that the insert is of very low concentration and it is in the form of discrete infinitesimal regions in which heat conduction is insignificant. The parameter ω represents the dimensionless angular velocity of the fluctuating heating source, and U_o represents the dimensionless amplitude of the heating source which generates heat in the matrix.

Figures 2–4 shows the harmonic variation in temperature as predicted by the three models at different values of the angular velocity ω. All three models show that this deviation increases as ω increases. This

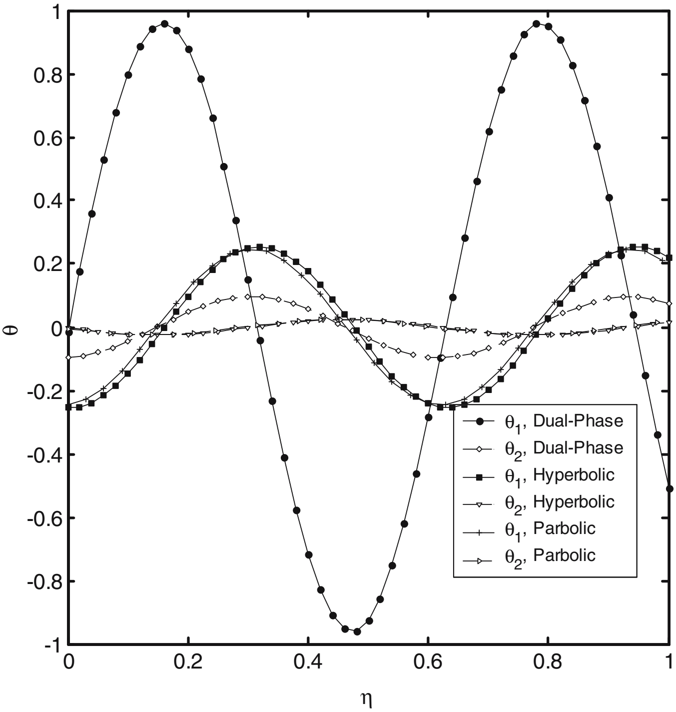

Fig. 4. Harmonic variation in temperature as predicted by the three models. ($\omega = 10$, $\xi = 0.5$, $\tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}$, $\tau_{q,1} = 1 \times 10^{-2}$, $E = F = 1$).

phenomenon can also be concluded by looking at Eqs. (18)–(22) which show that as ω increases β decreases, and as a result, the deviation between V_1 and V_2 increases and hence the deviation between θ_1 and θ_2 increases. This is a result of the fact that as ω increases, the heat is generated in domain 1 at a higher and higher frequency which does not give domain 2 enough time to attain the temperature of domain 1. This increases the deviation between the magnitudes of both domains' temperatures and also increases the phase shift between the two temperatures as will be shown later. The figures also show that for a large value of ω, the relative deviation between θ_1 and θ_2 increases but the absolute deviation decreases due to the reduction in the values of θ_1 and θ_2. The reduction in θ_1 and θ_2 with an increase in ω is due to the fact that both domains do not have enough time to absorb heat from the generating heating source when the heating source fluctuates with a high frequency. It may be concluded that the deviation between θ_1 and θ_2 appears at $\omega > 0.05$.

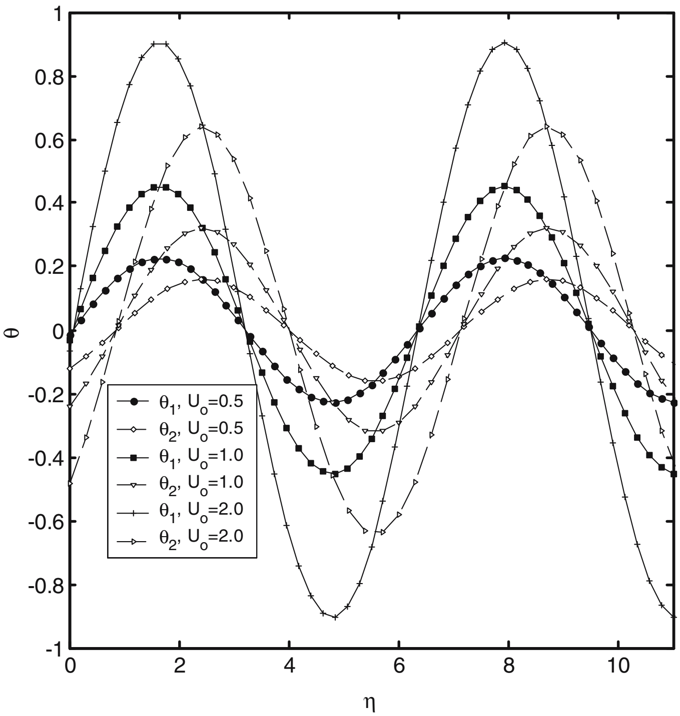

Fig. 5. Harmonic variation in temperature as predicted by the dual-phase-lag model at different values of U_o.($\omega = 1.0, \xi = 0.5, \tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}, \tau_{q,1} = 1 \times 10^{-2}, E = F = 1$).

Figures 2–4 also show that the deviation between the dual-phase-lag model and the other two models starts to appear at $\omega > 0.5$ and the deviation between the hyperbolic and parabolic models starts to appear at $\omega > 5$.

The phase lag between the heat flux q and the temperature gradient $\frac{\partial T}{\partial x}$ is relatively small, but this phase lag becomes a significant effect when the heat flux is generated with a very high frequency. This fact may be concluded by looking at Eq. (19). Also, Figs. 2–4 show that the deviation between the dual-phase-lag and parabolic heat-conduction models appears before the deviation between the parabolic and hyperbolic models. This is expected since the dual-phase-lag heat-conduction model takes into consideration two sources of deviation; the phase lag in heat flux (due to τ_q) and the phase lag in temperature gradient (due to τ_T). The phase shift between θ_1 and θ_2 increases as ω increases but this phase shift reaches the

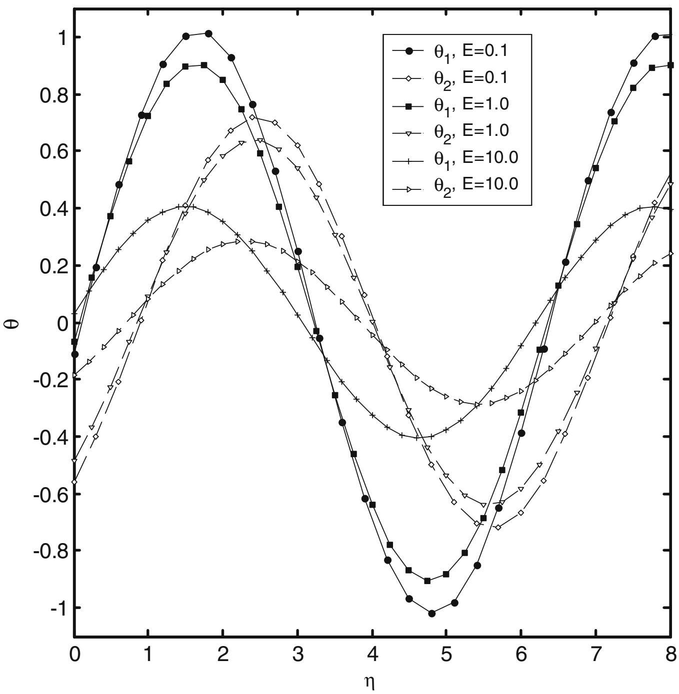

Fig. 6. Harmonic variation in temperature as predicted by the dual-phase-lag model at different values of E. ($\omega = 1.0, \xi = 0.5, \tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}, \tau_{q,1} = 1 \times 10^{-2}, F = 1$).

asymptotic value $-\frac{\pi}{2}$ as ω becomes very large. This behavior is predicted by all three heat-conduction models.

Figure 5 shows the effect of the dimensionless heating source U_o on the composite domain thermal behavior. It is clear from the figure that U_o affects the amplitude of the deviation between θ_1 and θ_2 but it has no effect on the phase shift between them. The figure shows that the deviation between θ_1 and θ_2 increases as U_o increases. The same is true for the hyperbolic heat-conduction model which is a special case obtained from the dual-phase-lag model with $\tau_{T,1} = 0$.

The effect of the modified volumetric heat transfer coefficients (E, F) on the composite domain thermal behavior under the effect of the dual-phase-lag heat-conduction model is shown in Figs. 6 and 7. As E and F increase, the convective heat transfer from the hot matrix to the insert increases, and as a result, θ_1 decreases and θ_2 increases. However, the reduction in θ_1 is insignificant when compared to the increase in θ_2. This is due to the relatively low mass fraction of the insert compared to

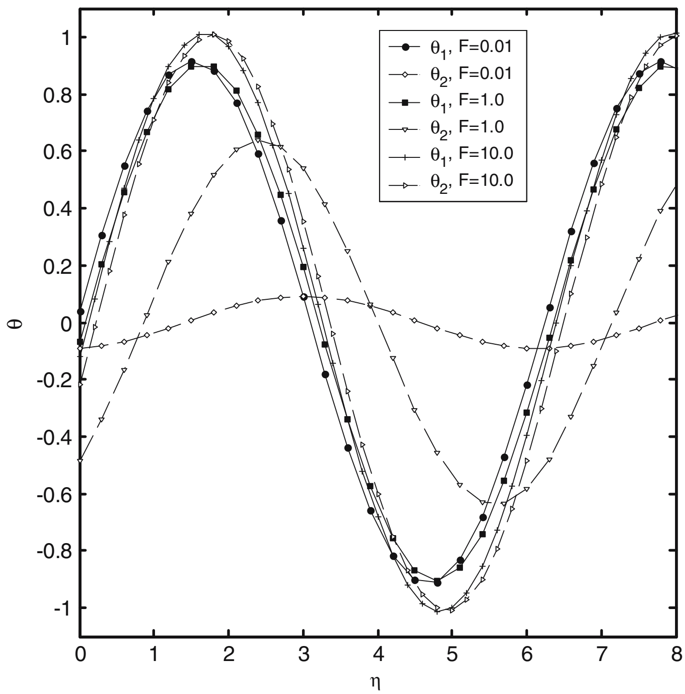

Fig. 7. Harmonic variation in temperature as predicted by the dual-phase-lag model at different values of F. ($\omega = 1.0, \xi = 0.5, \tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}, \tau\tau_{q,1} = 1 \times 10^{-2}, E = 1$).

the mass fraction of the matrix. The composite domain is assumed to have ε close to zero which yields inserts of low total thermal capacity. At large values of E and F, both domains attain almost the same temperature due to the increase in the convective heat transfer between them. Also, the phase shift between the two temperatures θ_1 and θ_2 diminishes as E and F increase.

The effect of E and F on θ_1 and θ_2 becomes insignificant at large values of E and F. As the convective heat transfer coefficients E and F increase, the heat transfer between θ_1 and θ_2 increases and θ_2 approaches θ_1. In the limiting case of E and F$\rightarrow \infty$, then $\theta_2 \rightarrow \theta_1$, and the convective heat transfer between θ_1 and θ_2, which is proportional to E $(\theta_1 - \theta_2)$ or F $(\theta_1 - \theta_2)$, diminishes. This implies that the effect of E and F disappears as $\theta_2 \rightarrow \theta_1$. The very same behavior holds true for the hyperbolic heat-conduction model.

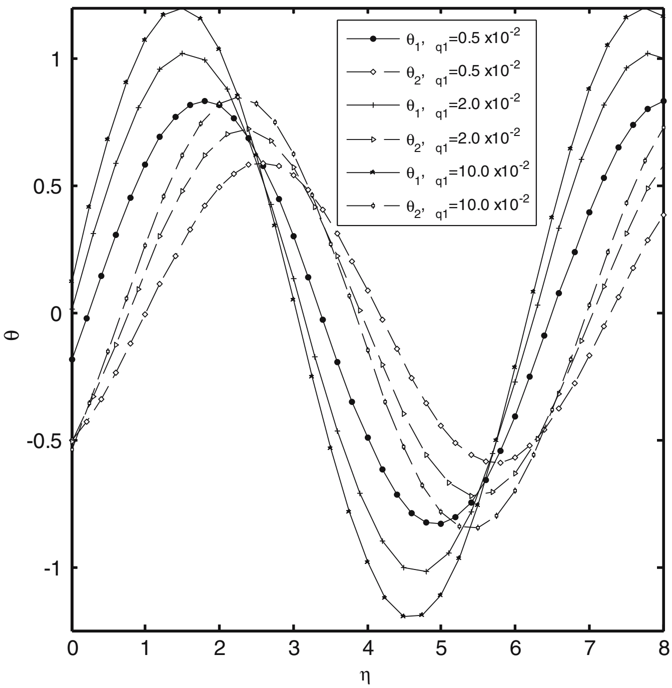

Fig. 8. Harmonic variation in temperature as predicted by the dual-phase-lag model at different values of $\tau_{q,1}$. ($\omega = .0, \xi = 0.5, \tau_{T,1} = \tau_{T,2} = 150\tau_{q,1}, E = F = 1$).

Figure 8 shows the harmonic variation in temperature as predicted by the dual-phase-lag model at different values of $\tau_{q,1}$. It is clear that for a large $\tau_{q,1}E; F$ has a more significant effect on θ_1 and $\theta 2$, and also the amplitude increase as $\tau_{q,1}$. Figure 9 shows the harmonic variation in temperature as predicted by the dual-phase-lag model at different values of $\tau_T, 1$.

The behavior of the phase shift δ with changing frequency ω is shown in Fig. 10. It is clear that the phase shift increases with an increase in the heating source frequency until it reaches the asymptotic value of $-1.57(-\pi/2)$, and is not effected by the variation of $\tau_{q,2}$.

5. CONCLUSIONS

In this study, the validity of using three macroscopic heat-conduction models under the effect of a fluctuating volumetric heating source, which heats a composite thin layer is investigated. The composite layer consists

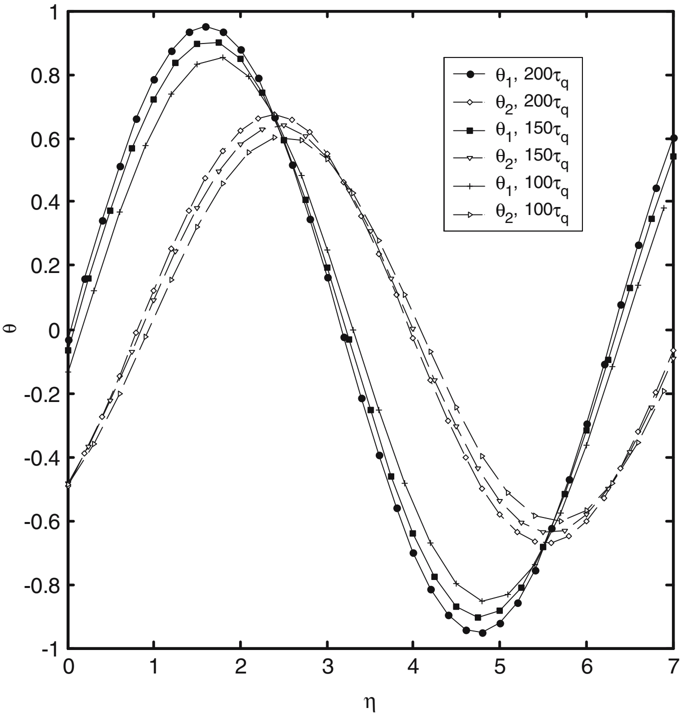

Fig. 9. Harmonic variation in temperature as predicted by the dual-phase-lag model at different values of $\tau_{T,2}$. ($\omega = 1.0$, $\xi = 0.5$, $\tau_{q,1} = 1 \times 10^{-2}$, $E = F = 1$).

of a matrix (domain 1) and inserts (domain 2); the inserts may be added to reinforce the slab's structure or to give the slab certain required physical and chemical properties. The three models are the parabolic, hyperbolic, and dual-phase-lag heat-conduction models. The heating disturbance is assumed to fluctuate in a harmonic manner. It is found that the use of the dual-phase-lag heat-conduction model is essential at high frequencies of the surface disturbance. It was found that the use of the dual-phase-lag heat-conduction model is essential at high frequencies of the volumetric disturbance ($\omega > 0.05$). It is found that as E and F (a kind of dimensionless convective heat transfer coefficients) increase, the difference between θ_1 and θ_2 decreases. An increase of the heating source magnitude U_o results in an increase of the magnitudes of θ_1 and θ_2 but does not affect the difference between them. As for the phase shift δ, it is found to increase as the frequency ω increases until it reaches the fixed asymptotic value of $-\frac{\pi}{2}$ at very high frequencies.

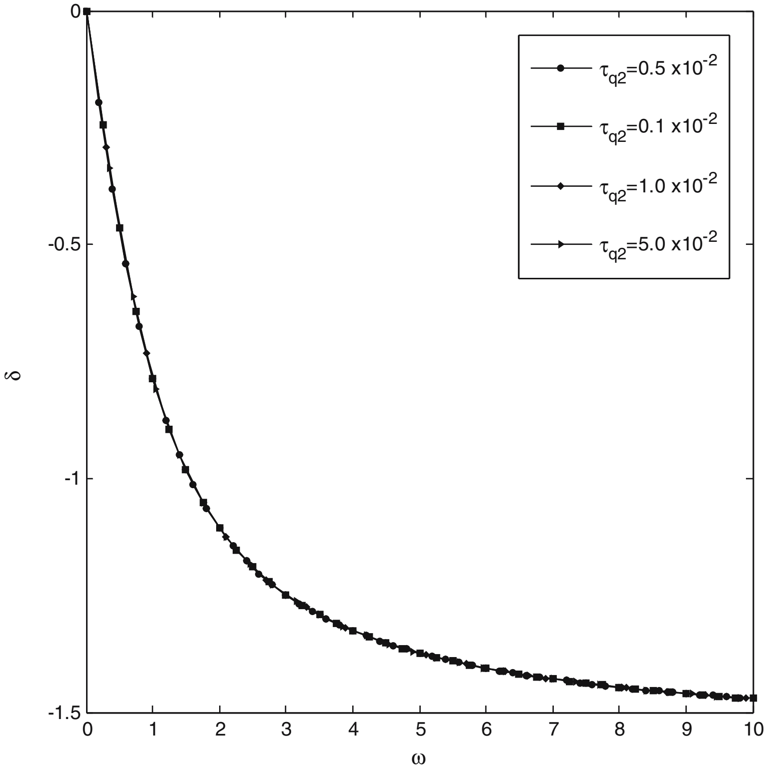

Fig. 10. Phase shift δ over a wide range of heating source frequency ω at different values of $\tau_{q,1}$.

NOMENCLATURE

c	specific heat
h	volumetric heat transfer coefficient
k	thermal conductivity
$2L$	slab thickness
t	time
T	temperature
T_{w}	wall temperature
u	heating source per unit volume
u_o	amplitude of the eating source per unit volume
U	dimensionless heating source, $\frac{u\alpha\overline{\tau}_q}{T_\infty k}$
U_o	amplitude of the dimensionless heating source, $\frac{u_o\alpha\overline{\tau}_q}{T_\infty k}$
x	axial coordinate

Greek Symbols

α	thermal diffusivity, $\frac{k}{\rho c}$
η	dimensionless time, $\frac{\alpha t}{L^2}$
ρ	density
ε	the insert (material 2) volume fraction, $\frac{V_2}{V_1+V_2}$
θ	dimensionless temperature, $\frac{T-T_\infty}{T_\infty}$
$\overline{\tau}_q$	phase lag in heat flux
τ_q	dimensionless phase lag in heat flux, $\frac{\alpha\overline{\tau}_q}{L^2}$
$\overline{\tau}_T$	phase lag in temperature gradient
τ_T	dimensionless phase lag in temperature gradient, $\frac{\alpha\overline{\tau}_T}{L^2}$
ξ	dimensionless axial coordinate, $\frac{x}{L}$
$\overline{\omega}$	angular velocity of the fluctuating wall temperature
ω	dimensionless angular velocity of the fluctuating wall temperature, $\frac{\overline{\omega}L^2}{\alpha}$

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